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some process of maturing, while the process itself might be vague and hence undefinable. Judging from the writings which are now commonly classed under mathematical history it would appear that the chronological element was generally considered an essential element in a direct historical paper. In an indirect historical paper,—for instance, one discussing how mathematical history should be written—this element would not need to be present.

Next in importance to the chronological element in the usual historical presentation of a mathematical subject is the human element. Even the mere names of those who have enriched mathematical thought by pointing out logical steps leading to views of unusual beauty or to regions of unusual fruitfulness serve to establish a sense of comradeship. This sense is intensified by more or less complete biographical notices. It should, however, be observed that these names and these biographical sketches serve other useful purposes. The names of great mathematicians may be used to unify varied mathematical results, while the biographical sketches serve to unify mathematical knowledge and knowledge relating to other lines of thought.

While these considerations may throw some light on the term history of mathematics they are not intended to convey the idea that this term could be defined in a perfectly satisfactory manner. Like some other useful mathematical terms, the term history of mathematics will probably always remain without a real definition. While one may not know where mathematical history begins, yet there are some writings which all agree to classify under history and others which few would classify under this heading. On the other hand, there seems to be a strong tendency towards increasing the historical element in modern mathematical writings so that the writings which are zero per cent. history are becoming less and less common. It is to be hoped that the mathematical teaching which devotes zero per cent. of the time to history, and the mathematical history which includes zero per cent. of the present-day mathematical activities will also become less and less common.

ON THE CIRCLES OF APOLLONIUS.

(*Concluded*)

By NATHAN ALTSHILLER, University of Colorado.

(By some unaccountable oversight this last page of DR. ALTSHILLER'S article was omitted from the October issue. EDITORS.)

12. The three pairs of points A, A'; B, B'; C, C' of ω are perspective from K; they belong therefore to the same involution on ω, K and l being respectively the pole and the axis of the involution[1] (9A). Hence the couples of lines AB, $A'B'$; BC, $B'C'$; AC, $A'C'$ meet on l. Now, C_3, C_1, C_2 are the points common to l and the lines AB, BC, AC respectively. Hence:

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[1] Russell, *loc. cit.*, pp. 217, 218.

The lines $A'B'$, $B'C'$, $A'C'$ meet the lines AB, BC, AC respectively in the centers of the Apollonian Circles.

13. The points A, B, A', B' are the vertices of a complete quadrangle inscribed in ω. The polars s_3 and l, with respect to ω (8, 9), of the diagonal points $C_3 \equiv (AB, A'B')$ (12) and $K \equiv (AA', BB')$ (7) intersect in the third diagonal point[1] $(AB', A'B)$, *i. e.*, this point is identical with P_3 (11). Similarly for the poles P_1 and P_2. Hence:

The pairs of lines AB', $A'B$; BC' $B'C$; AC', $A'C$ meet respectively in the poles P_3, P_1, P_2 of the Brocard diameter with regard to the Apollonian circles.

14. The triangles KP_1C_1, KP_2C_2, KP_3C_3, are self-conjugate with respect to ω (13), and therefore:

The center of an Apollonian circle and the pole, with regard to this circle, of the Brocard diameter are conjugate points with respect to the circumcircle.

15. The involution of conjugate points P_1, C_1; P_2, C_2; P_3, C_3 with respect to ω (14) is the section of the orthogonal involution of rays having its center at I; the rays joining I to the two points of intersection of 1 with any one of the Apollonian Circles are conjugate in this involution. Consequently:

The points of intersection of the Lemoine line with an Apollonian circle are conjugate with respect to the circumcircle.

16. Lemma. *If two tangents to two circles meet on the radical axis, the points of contact are collinear with one of the centers of similitude of the circles.*

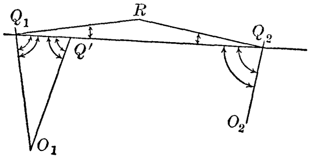

FIG. 2.

The points of contact Q_1, Q_2 and the point of intersection R of the tangents form an isosceles triangle, since $RQ_1 = RQ_2$. O_1 and O_2 being the centers of the circles, the angles $O_1Q_1Q_2$ and $O_2Q_2Q_1$ are equal, being the respective complements of two equal angles. Now, if Q' is the second point common to Q_1Q_2 and the circle O_1, we have $\angle O_1Q'Q_1 = \angle O_1Q_1Q_2 = \angle O_2Q_2Q_1$; hence O_1Q' is parallel to O_2Q_2, which proves the lemma.

This proof changes but slightly if O_1, O_2 are on opposite sides of Q_1Q_2.

17. The line $A'B'$ (Fig. 1) joining the points of contact A', B' of the two tangents OA', OB' to the two circles γ_1, γ_2, meets the line of their centers l in one of their centers of similitude (16). This is the point C_3 (12). The line AB' joining the points of contact A, B' of the two tangents OA, OB' to the two circles γ_1, γ_2 meets the line l in one of their centers of similitude (16). This is the point P_3 (13). Similarly for the points C_1, P_1, C_2, P_2. Hence:

The center of an Apollonian circle and the pole, with respect to this circle, of the Brocard diameter, are the two centers of similitude of the two other circles of Apollonius.

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[1] Russell, *loc. cit.*, pp. 34, 35.